Examinee No.	

(Fill in your examinee number above)

2024 Academic Year

The University of Tokyo, Graduate School of Engineering

Entrance Examination

for

Department of Aeronautics and Astronautics

Morning Session

(9:00～12:00)

IMPORTANT NOTICES

1. Do not open this booklet before the start of the examination.
2. Choose and answer the questions from three fields out of four.
3. Three answer sheets are given. Use each sheet to answer the problems of different field.
4. Fill the field name and your examinee number in the answer sheet.
5. Do not carry out this booklet nor the answer sheets.

Fluid Dynamics (Morning)

Consider the turbulent boundary layer formed on an object in a two-dimensional steady incompressible viscous flow with the constant viscosity μ and the constant density ρ. Use the coordinates where the x coordinate is along the surface of the object and the y coordinate is normal to it, and the origin of y coordinate is on the surface of the object. u is the velocity component in the x direction and v is the one in the y direction. U is the velocity component in the x direction at the boundary layer edge. δ is the boundary layer thickness. τ is the shearing stress acting on the fluids. τ_w is the shearing stress at the surface of the object. The boundary layer equation is expressed as eq. (1)

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\mathrm{d}U}{\mathrm{d}x}+\frac{1}{\rho}\frac{\partial \tau}{\partial y}. \tag{1}$$

The displacement thickness δ^* and the momentum thickness θ are defined as eqs. (2). They are used as parameters that express the boundary layer thickness. The shape factor H is defined as eq. (3). Then it is known eq. (4) holds.

$$\delta^* = \int_0^\delta (1-\frac{u}{U})\mathrm{d}y, \quad \theta = \int_0^\delta \frac{u}{U}(1-\frac{u}{U})\mathrm{d}y \tag{2}$$

$$H=\frac{\delta^*}{\theta} \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(U^2\theta)+\delta^* U\frac{\mathrm{d}U}{\mathrm{d}x}=\frac{\tau_w}{\rho} \tag{4}$$

Here, we consider the case when U is the function of only x. We discuss the turbulent boundary layer developing along x. Answer the following questions.

Question 1

Past research confirms experimentally that τ_w of the turbulent boundary layer is expressed by eq. (5).

$$\frac{\tau_w}{\rho U^2}=AR^{-p}, \tag{5}$$

where A is the function of H, R is the Reynolds number defined by eq. (6) and p is the positive constant

$$R=\frac{\rho U\theta}{\mu}. \tag{6}$$

Show eq. (7) holds.

$$\frac{\mathrm{d}z}{\mathrm{d}x}+\frac{k}{U}\frac{\mathrm{d}U}{\mathrm{d}x}z-B=0, \tag{7}$$

where

$$z=\theta R^p, \quad B=A(1+p), \quad k=(1+p)(2+H)-p.$$

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Question 2

We consider the case when the boundary layer characteristics does not change significantly towards the downstream direction. We consider H does not change along x. By using eq. (7), show eq. (8) holds. Using eq. (8), the changes in the momentum thickness along x can be obtained.

$$zU^k = z_0 (U_0)^k + B\int_{x_0}^{x} U^k \mathrm{d}x , \tag{8}$$

where the subscript 0 means the physical quantity at the x position which is used as a reference point on the object.

Question 3

In the previous question, we consider H does not change along x. In reality, H changes. To consider this effect, we define the energy thickness θ^* as one of the parameters indicating the boundary layer thickness as shown in eq. (9)

$$\theta^* = \int_0^{\delta} \frac{u}{U}(1-\frac{u^2}{U^2})\mathrm{d}y . \tag{9}$$

Derive the following relation shown in eq. (10), by multiplying u on both sides of eq. (1) and by integrating it in the y direction from $y=0$ to $y=h$, where h is a constant and $y=h$ is outside the boundary layer.

$$\frac{\mathrm{d}}{\mathrm{d}x}(U^3\theta^*) = 2U^3E , \tag{10}$$

where

$$E = \int_0^{\delta} \frac{\tau}{\rho U^2}\frac{\partial}{\partial y}(\frac{u}{U})\mathrm{d}y . \tag{11}$$

You may use the formula shown in eq. (12) assuming a and b are constants

$$\int_a^b \frac{\partial}{\partial x} f(x,y)\mathrm{d}y = \frac{\mathrm{d}}{\mathrm{d}x}\int_a^b f(x,y)\mathrm{d}y . \tag{12}$$

Question 4

Derive eq. (13) which expresses the change in H along x by deleting the derivative of θ with respect to x using eqs. (4) and (10).

$$z\frac{\mathrm{d}H}{\mathrm{d}x} + (G\frac{\tau_w}{\rho U^2} - 2E)R^p\frac{\mathrm{d}H}{\mathrm{d}G} - (H-1)G\frac{z}{U}\frac{\mathrm{d}H}{\mathrm{d}G}\frac{\mathrm{d}U}{\mathrm{d}x} = 0 , \tag{13}$$

where G is the dimensionless quantity defined as eq. (14) from the analogy of H.

$$G = \frac{\theta^*}{\theta} \tag{14}$$

Solid Mechanics (Morning)

Consider a mast and two flexible solar array panels supported by the mast in Figure 1. As shown in Figure 2, the mast is modeled as a uniform cantilevered beam, and the two panels are collectively modeled as single uniform string of which each end is tied to each end of the beam. Then, no offset at each end between the beam and the string exists in all three axes direction. Bending deflection $w(x)$ of the beam only in xz plane is considered. As the coefficient of thermal expansion of the beam α is larger than that of the string α_0 $(\alpha > \alpha_0 > 0)$, the string of its equilibrium length L is tied to the beam of its equilibrium length $L + \delta$ at the normal temperature T_N so that the tension of the string just becomes zero at the low temperature T_L $(T_L < T_N)$. Moreover, the cross-sectional area of the beam is determined so that the compressive load of the beam at the high temperature T_H $(T_H > T_N)$ just comes up to Euler buckling load as shown in Figure 3. The cross-sectional area, Young's modulus and area moment of inertia of the beam are denoted by A, E and I, respectively. The cross-sectional area and Young's modulus of the string are denoted by A_0 and E_0, respectively. The temperatures of the string and the beam are the same. And it is assumed that the string has no bending stiffness and is not able to transmit compressive load. Answer the following questions. If necessary, use the following approximations.

$$L \gg \delta, \quad 1 \gg \alpha(T_H - T_L) \tag{1}$$

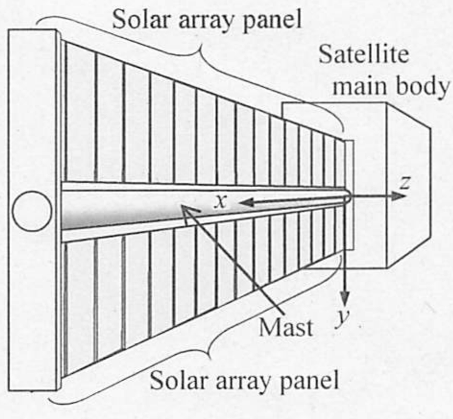

Fig.1 Mast and solar array panels

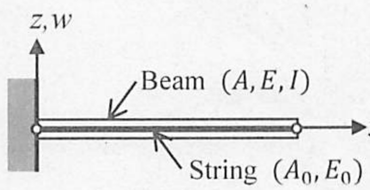

Fig.2 Beam and string model

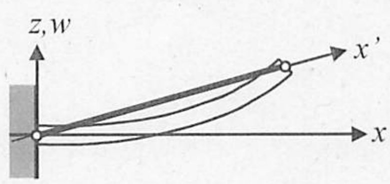

Fig.3 Bending deflection after Euler buckling

Question 1

Assuming that the compressive load P yields the bending deflection $w(x)$, derive the differential equation of Euler buckling from the equilibrium of the loads on an infinitesimal element in the bended beam. Include the figure of the infinitesimal element and the compressive load P, shearing load Q and bending moment M working on it and the derivation procedure of the equation in the answer. If necessary, use the following relation between M and $w(x)$.

$$M = -EI\frac{d^2w}{dx^2} \tag{2}$$

Question 2

In case of using the bending deflection w expressed by x'-coordinate shown in Figure 3 instead of x-coordinate in order to obtain Euler buckling load of this beam, select the boundary conditions applied to its tip and root from "Fixed", "Simply Supported", "Free" conditions, respectively. And describe the reason of your selection.

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Question 3

Express the length δ in the equilibrium length $L+\delta$ of the beam, using L, α, α_0, T_N and T_L.

Question 4

Express the magnitude of the compressive load P_0 generated in the beam at the high temperature T_H, using $\alpha, \alpha_0, T_H, T_L$ and so on.

A mast designed according to the aforementioned approach buckled in *xz* plane at the low temperature T_L on orbit. The results of investigation revealed the followings.

- The load of the mast at the low temperature T_L came up to the compressive load P_0 obtained in Question 4.
- Young's modulus of a material X used in the solar array panels sharply changes around the temperature T_g $(T_N > T_g > T_L)$.
- In the design process, the change of Young's modulus of the material X was overlooked, and it was decided that the structural member made of the material X didn't need to be reflected in the string model of the solar array panels.

Instead of reflecting the structural member made of the material X in the existing string, it is newly modeled as another uniform string of the equilibrium length L at the normal temperature T_N. This string is added to the model in Figure 2, and its each end is tied to each end of the beam as well as the existing string. Consequently, the modified model in Figure 2 consists of the beam and the two strings. The cross-sectional area, Young's modulus and coefficient of thermal expansion of the new string are denoted by A_1, E_1 and α_1. Young's modulus E_1 of the string is 1.0% of Young's modulus E of the beam at the normal temperature T_N.

Question 5

Obtain the ratio E_1/E at the low temperature T_L, using the following relations.

$$T_N = 300 \text{ [K]},\ T_L = 100 \text{ [K]},\ T_H = 400 \text{ [K]}$$

$$E_0 = 0.1E,\ A_0 = A,\ A_1 = 0.01A,\ \alpha = 5\alpha_0,\ \alpha_1 = 9\alpha_0$$

And describe the reason why the mast buckled at the low temperature, including the reason why it was decided in the design process that the structural member made of the material X didn't need to be reflected in the string model.

Aerospace System (Morning)

Consider the attitude control during ballistic flight for a rocket flying from the atmosphere to outer space shown in Figure 1. The principal axes of the rocket coincide with the body coordinate axes, and the moment of inertia about X, Y, and Z axes are I_{xx}, I_{yy}, and I_{zz} (>0, constant), the body angular velocities are ω_x, ω_y, and ω_z, the control torques are N_x, N_y, and N_z in the body coordinate system, and the time is t. Assume the rocket to be a rigid body. Answer the following questions. If any assumptions, variables, or constants are needed to answer, define them before using.

Question 1

Derive the equation of motion for the rotational motion (Euler equation) in the body coordinate system for each of the X, Y, and Z axes using the moment of inertia, body angular velocities, and control torques. Atmospheric effects are not considered here.

Question 2

Assuming that the angular velocities and the control torques are $\omega_y=\omega_z=0$, $N_y=N_z=0$, we consider the spin control around the X axis when the rocket flies in the atmosphere. Assuming $I_{yy}=I_{zz}=I$, and the angular velocity damping coefficient around the X axis due to atmospheric effects is c_x (>0, constant), the equation of motion about the X axis is as follows.

$$I_{xx}\frac{d\omega_x}{dt}+c_x\omega_x=N_x \tag{1}$$

The target value of the angular velocity ω_x is r, and the deviation is $e=r-\omega_x$.

1. Laplace transforms of N_x, ω_x, and r are $N(s)$, $W(s)$, and $R(s)$. Express the transfer function $P(s)$ from $N(s)$ to $W(s)$ using c_x and I_{xx}.
2. The feedback control according to $N_x=k_pe$ with proportional gain k_p is applied as the spin control around the X axis. Draw a block diagram for this control system using $N(s)$, $W(s)$, $R(s)$, $P(s)$, and k_p. And express the transfer function $T(s)$ from $R(s)$ to $W(s)$ using k_p, c_x and I_{xx}.
3. Find the condition of k_p for this feedback control system to be stable.
4. It is assumed that k_p satisfies the stability condition of this control system. For this control system, consider a rate following control by a step input $r(t)=bu(t)$ as a target value (b : positive constant, $u(t)$: unit step input). When the control time is sufficiently long, obtain the steady-state deviation e using the final value theorem.

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Question 3

After the completion of spin control, the rocket escaped from the atmosphere at time $t = t_0$ after flying for a while. It is assumed that $N_x = N_y = N_z = 0$, and the following attitude motion (nutation motion) was observed after escaping the atmosphere.

$$\begin{cases} \omega_x = \omega_s \\ \omega_y = \omega_p \sin\{n(t - t_0)\} \\ \omega_z = \omega_p \cos\{n(t - t_0)\} \end{cases} \tag{2}$$

$\omega_s(> 0)$, $\omega_p(> 0)$, n are all constants. Assume $I_{yy} = I_{zz} = I$, and $I > I_{xx}$. Atmospheric effects are not considered here.

1. Express n by ω_s, I, and I_{xx} using the equation of motion derived in question 1.
2. Assume that the angle (nutation angle) between the angular momentum vector and the X-axis is α shown in Figure 2. When the observed nutation motion is $\omega_p = 0.025\omega_s$ and $\alpha = 0.2$ rad, obtain I/I_{xx}. The approximation $\tan\alpha \approx \alpha$ can be used.

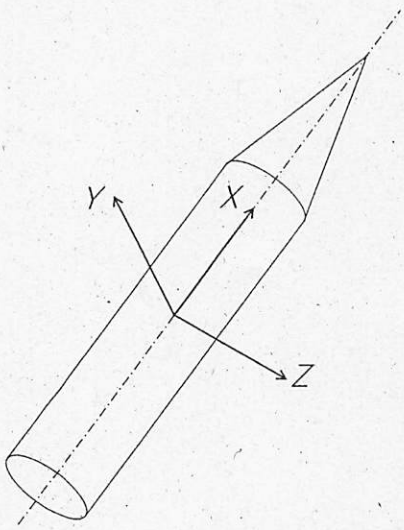

Figure 1

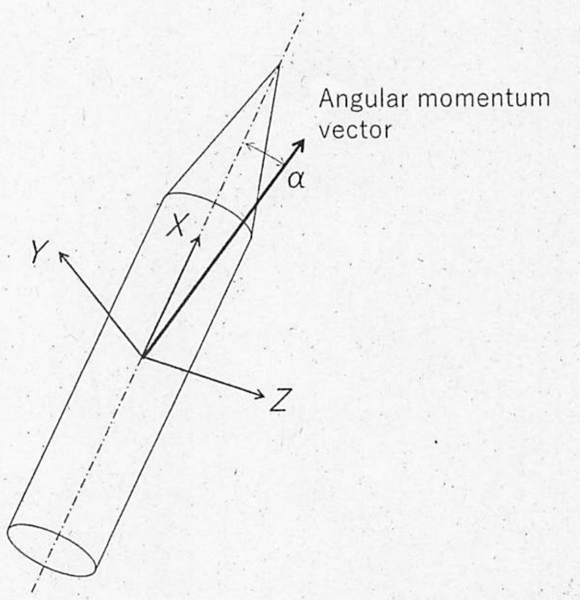

Figure 2

Propulsion Engineering (Morning)

The principle of the vibrating structure gyroscope is that when an angular velocity Ω_z around the z-axis perpendicular to the paper surface is applied to the box as shown in Figure 1, the Coriolis force acting on the two-degree-of-freedom oscillator in the $x-y$ plane inside the box is detected. Here, the coordinate system is fixed to the box, and its origin is at the center of the box. Angular velocity Ω_z can be estimated from the amplitude of vibration generated in the orthogonal direction (y-axis) by driving the oscillator with excitation force along the x-axis in Figure 1. Note that the oscillator can be considered as a point mass.

Question 1
Show the equations of motion of this oscillator. Here, use m, C_i, k_i, f_i as the oscillator mass, damping constant, spring constant, and excitation force, respectively. Note that $i = x, y$, and the same applies hereinafter.

Question 2
Rewrite the equations of motion for Question 1 using the natural angular frequency ω_i of the undamped system and the vibrational quality factor $Q_i = \sqrt{mk_i}/C_i$.

Do not use C_i, k_i in subsequent answers, use ω_i, Q_i instead.

Question 3
Consider the case where $\Omega_z = 0$. In the x-axis equation of motion, find the condition of the Q_x and the angular frequency ω when the x-axis vibration takes the maximum amplitude.

Question 4
Consider the case where an angular velocity Ω_z is applied. Assume that the oscillator is driven in the x-axis direction with amplitude X_0 and frequency $\omega = \omega_x$ as $x = X_0 \sin(\omega_x t)$ where t is time. Solve the y-axis motion equation with $f_y = 0$, and find the relationship between the y-axis amplitude Y_0 and Ω_z in this case.

Question 5
A gyroscope that directly measures the rotation angle without numerically integrating the angular velocity is called an integrating gyroscope. A Foucault pendulum has a similar principle. Consider the requirements and operations required when trying to realize something equivalent with the two-degree-of-freedom oscillator as shown in Figure 1.

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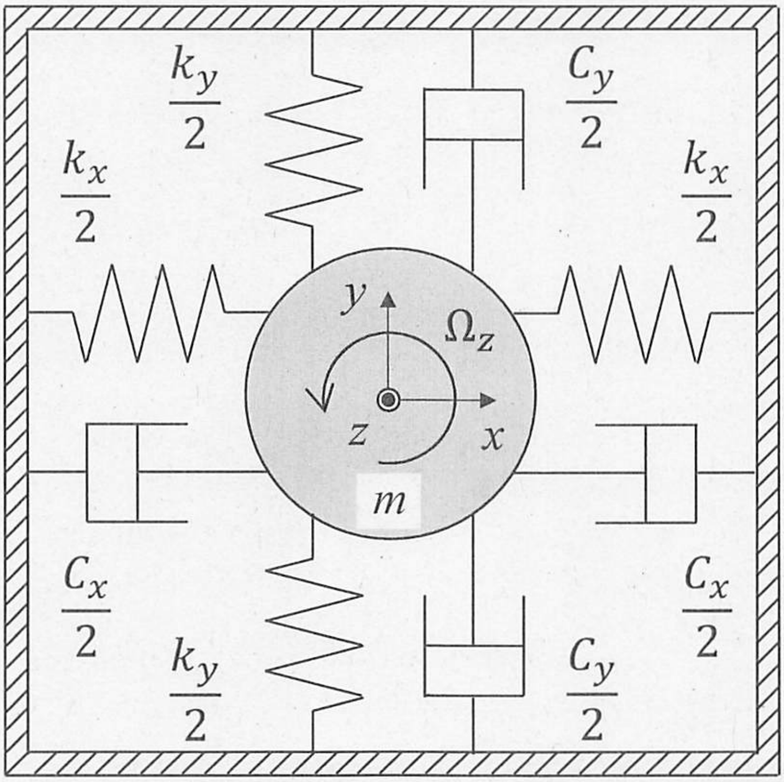

Figure 1 Two-degree-of-freedom oscillator in a box.