Examinee No.	

(Fill in your examinee number above)

2024 Academic Year

The University of Tokyo, Graduate School of Engineering

Entrance Examination

for

Department of Aeronautics and Astronautics

Afternoon Session

(13:30～16:30)

IMPORTANT NOTICES

1. Do not open this booklet before the start of the examination.
2. Choose and answer the questions from three fields out of four.
3. Three answer sheets are given. Use each sheet to answer the problems of different field.
4. Fill the field name and your examinee number in the answer sheet.
5. Do not carry out this booklet nor the answer sheets.

Fluid Dynamics (Afternoon)

Consider two-dimensional inviscid compressible flow around an airfoil as a model for a steady transonic flow around the main wing of an airplane flying at constant altitude through the atmosphere at rest. Assume that the atmosphere is calorically perfect gas, and γ is the specific heat ratio.

Question 1

When the flow speed of a point on the airfoil becomes exactly the speed of sound, the flight Mach number is M_{A} (critical Mach number). Obtain the pressure coefficient C_P^* at the point of lowest pressure on the airfoil using M_{A} and γ. Assume that no separation occurs on the airfoil, and the definition of pressure coefficient C_P in terms of pressure P is given by the following equation,

$$C_P = \frac{P - P_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} \, . \tag{1}$$

Note that uniform flow density, pressure and velocity are ρ_∞, P_∞, and U_∞, respectively.

Next, consider a steady shock wave generated around the airfoil. The flight Mach number M_{B} satisfies $M_{\mathrm{A}} < M_{\mathrm{B}} < 1$ (No detached shock wave is generated).

Question 2

Draw an airfoil suitable for transonic flight on your answer sheet. Show the uniform flow direction by an arrow and then indicate the supersonic flow region with shaded lines. Also, indicate the position of the shock wave in the same diagram.

In the following, assume that a steady shock wave over the airfoil is a normal shock wave, and the flow is one-dimensional flow.

Question 3

When the airfoil flying at transonic speed U_{B} in the atmosphere at rest was observed from ground-based stationary frame, the flow speed upstream of the shock wave was u_{u}, and the downstream of the shock wave was atmosphere at rest (Consider proper directions for U_{B} and u_{u}). Show the equation of continuity, equation of momentum, and equation of energy, that are valid when a thin test volume surrounding the shock wave is considered in shock stationary frame. Note that density and pressure upstream of the shock wave are ρ_{u} and P_{u}, respectively. The density and pressure downstream of the shock wave are ρ_∞ and P_∞, respectively.

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Question 4

Obtain the value $u_{\mathrm{u}}/a_{\infty}$, which is the ratio of u_{u} to speed of the sound of the atmosphere at rest (a_{∞}), using flight Mach number $M_{\mathrm{B}}(= U_{\mathrm{B}}/a_{\infty})$ and γ.

Question 5

Obtain the value $P_{\mathrm{u}}/P_{\infty}$, which is the ratio of P_{u} to atmospheric pressure P_{∞}, using M_{B} and γ.

Question 6

The pressure coefficient at the upstream of the shock wave was $C_{P\mathrm{u}}$. Obtain the flight Mach number M_{B}.

Solid Mechanics (Afternoon)

The thin rectangular panel of size $L \times h$, reinforced with three rods attached to its edges as stiffeners, is fixed to the rigid wall at $x = 0$ as shown in Figure 1. All three rods can only carry axial force and have equal and uniform cross-sectional area A and Young's modulus E. These rods are pinned to the rigid wall and are mutually pin-connected. The thin panel is of uniform thickness t and has shear elasticity constant of G, and is perfectly joined to the rigid wall and the rods. Assuming that the deformation of the structure is confined within the plane of the thin panel, let us consider the case in which the force P acts at point C in the vertical direction ($-y$ direction).

Here, in the thin panel, normal stresses $\sigma_x = \sigma_y = 0$ and shear stress $\tau_{xy} \equiv \tau$ is assumed to be uniform. The dimensions of the rods in their thickness directions are presumed to be sufficiently small compared to the size h. Answer the following questions, using the values shown in the figure.

Question 1 Show the shear stress τ of the thin panel, and the axial force distributions $S_1(y)$, $S_2(x)$ and $S_3(x)$ of the rods 1, 2 and 3, respectively. Positive direction of τ is defined as shown in the figure and the axial tensile force is defined to be positive.

Question 2 Obtain the strain energy U stored in the structure.

Question 3 Obtain the displacement δ of point C in the loading direction using the Castigliano's theorem.

Question 4 The structure is considered as a cantilevered beam when L is sufficiently larger than h. Obtain the vertical displacement δ_{BE} at the free edge ($x = L$) of the beam, assuming the Bernoulli-Euler's hypothesis.

Question 5 Comparing the answers of above questions 3 and 4, determine the elasticity conditions which are to be enforced to the members composing the structure, so that the latter answer could be derived from the former answer.

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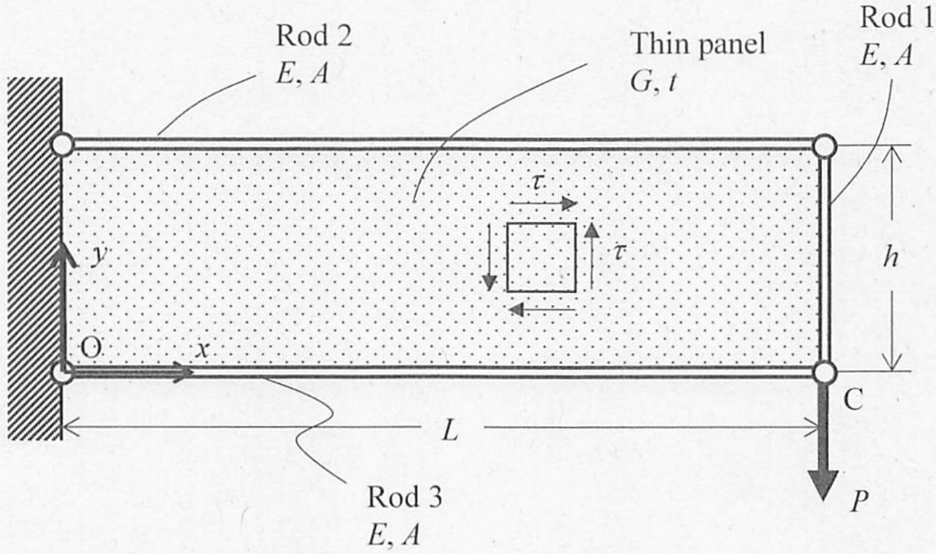

Figure 1

Aerospace System (Afternoon)

There is a dumbbell-shaped object composed of two uniform spheres with radius a and mass m connected by a mass-less rod, as shown in Figure 1. The object can be regarded as an axisymmetric rigid body. The distance between the centers of two spheres is l $(l > 2a)$. $(\dot{\ })$ means its time derivative. Answer the following questions.

Question 1 Express the moments of inertia of this object I_x, I_y, I_z around the three principal axes of inertia using m, l, a. Note that $I_x = I_y \equiv I_c$ holds, because of the axial symmetry. You may use the fact that the moment of inertia of a sphere with mass m and radius a about an axis passing through its center of mass is $\frac{2}{5}ma^2$.

Question 2 As shown in Figure 2, the object oscillates with small amplitude with a pivot point at a distance b $(b < \frac{1}{2}l - a)$ from its center of mass along the axis on the ground. Find the angular frequency ω_b of the oscillation. Let g denote the acceleration due to gravity on the surface of the Earth.

Next, we consider the attitude motion of the object in a zero-gravity environment as shown in Figure 3. When the angular velocity vector about the principal axes is denoted by $\boldsymbol{\omega} = [\omega_x,\ \omega_y,\ \omega_z]^{\mathrm{T}}$, the equation of motion in the principal axes coordinates is given by,

$$\boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega} = \boldsymbol{t} \ ,$$

where $\boldsymbol{I} = \begin{bmatrix} I_c & 0 & 0 \\ 0 & I_c & 0 \\ 0 & 0 & I_z \end{bmatrix}$ denotes the tensor of inertia, and $\boldsymbol{t} = [t_x, t_y, t_z]^{\mathrm{T}}$ denotes the torque vector due to the external forces.

Question 3 Show that the angular velocity about z-axis ω_z is constant, if $\boldsymbol{t} = \boldsymbol{0}$.

Question 4 Given $\boldsymbol{t} = \boldsymbol{0}$, letting the constant angular velocity about z-axis be denoted by Ω, describe how the angular velocities about x- and y-axes ω_x, ω_y change over time.

Question 5 Assume that an external torque $t_x = k\omega_x$ is applied to the object $(t_y = t_z = 0)$. Find the value of feedback gain k for which the oscillations of ω_x and ω_y are critically damped.

Finally, this object was launched into a circular orbit around the Earth as in Figure 4. Let the orbit radius of the center of mass and orbital angular velocity be denoted by r $(r \gg l)$ and ω_0 , respectively.

Question 6 Show that the attitude with z-axis of the object directed toward the center of the Earth is stable. Assume that the angular velocity about z-axis ω_z is zero.

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Question 7　Assume the object oscillates <u>in the orbital plane</u> with a small amplitude around the stable state given in the previous question. Find the angular frequency.

Question 8　Consider the orbital frame $\mathcal{F}_\mathrm{O} = \{X_\mathrm{O}, Y_\mathrm{O}, Z_\mathrm{O}\}$ with its origin fixed at the center of mass of the object, Z_O-axis directed toward the Earth center, and X_O-axis aligned with the flight direction of the object. Letting φ and ψ denote the roll and yaw angles, respectively, small attitude motions of the object about X_O- and Z_O- axes in the vicinity of $\varphi = \psi = 0$ are linearized as the following equations. Evaluate the stability of the system.

$$I_\mathrm{c}\ddot{\varphi} - \omega_0 I_\mathrm{z}\dot{\psi} + 4\omega_0^2 (I_\mathrm{c} - I_\mathrm{z})\varphi = 0 \ ,$$
$$I_\mathrm{z}\ddot{\psi} + \omega_0 I_\mathrm{z}\dot{\varphi} = 0 \ .$$

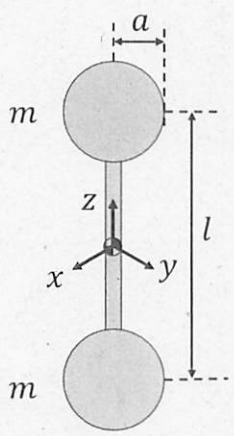

Figure 1

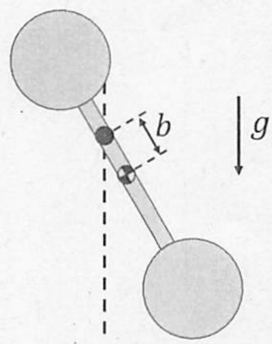

Figure 2

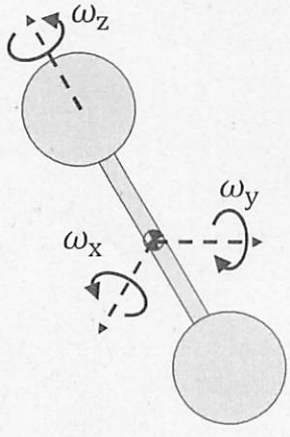

Figure 3

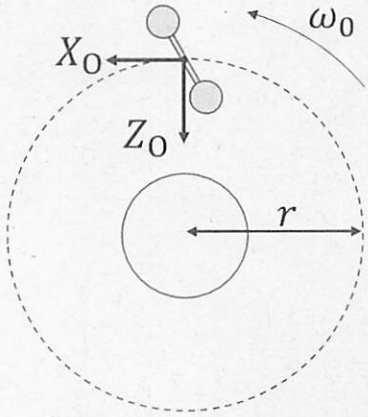

Figure 4

Propulsion Engineering (Afternoon)

Figure 1 shows the $p - v$ diagram (1→2→3→4→5→1, cycle A) of ideal cycle for a spark ignition engine whose expansion stroke is longer than the compression stroke. The pressure, temperature and specific volume at the condition i are defined as p_i, T_i and v_i, respectively. The compression ratio, expansion ratio and expansion-compression ratio of this cycle are defined as $\varepsilon_c = v_1/v_2$, $\varepsilon_e = v_4/v_2$ and $\varepsilon = \varepsilon_e/\varepsilon_c$, respectively. The pressure ratio of the isochoric heating process is defined as $\beta = p_3/p_2$. The operating fluid is assumed to be an ideal gas with gas constant R and specific heat ratio κ. Answer the following questions.

Question 1　Express p_2, p_3 in terms of $p_1, \beta, \varepsilon_c$ and κ.

Question 2　Express T_2, T_3 in terms of $T_1, \beta, \varepsilon_c$ and κ.

Question 3　Express T_4, T_5 in terms of T_1, β, ε and κ.

Question 4　Express the heat input per unit mass q_{1v} which the operating fluid receives at the isochoric heating process in terms of $R, T_1, \beta, \varepsilon_c$ and κ.

Question 5　Express the heat rejection per unit mass q_{2v}, q_{2p} which the operating fluid releases at the isochoric and the isobaric cooling processes in terms of $R, T_1, \beta, \varepsilon$ and κ, respectively.

Question 6　When the theoretical thermal efficiency η for this cycle is expressed as Eq. (1), express α in terms of β, ε and κ.

$$\eta = 1 - \frac{1}{\varepsilon_c^{\kappa-1}} \cdot \alpha \quad (1)$$

Question 7　In the case of the cycle in which the pressure at the end of adiabatic expansion process is equal to the initial pressure p_1, such as the cycle (1→2→3→6→1, cycle B) in Fig. 1, express the expansion-compression ratio ε in terms of β, κ. Then, express α in Eq. (1) in terms of β, κ.

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Question 8 Overlay the schematic of $T-s$ diagrams for cycles A and B. Also overlay the schematic diagram of $T-s$ diagram for an Otto cycle with the same compression ratio ε_c and the pressure ratio β, starting from the condition 1. Explain qualitatively the relationship between the theoretical thermal efficiencies of these three cycles, based on the diagrams. Here, s is the specific entropy, and the operating fluid are the same for these three cycles.

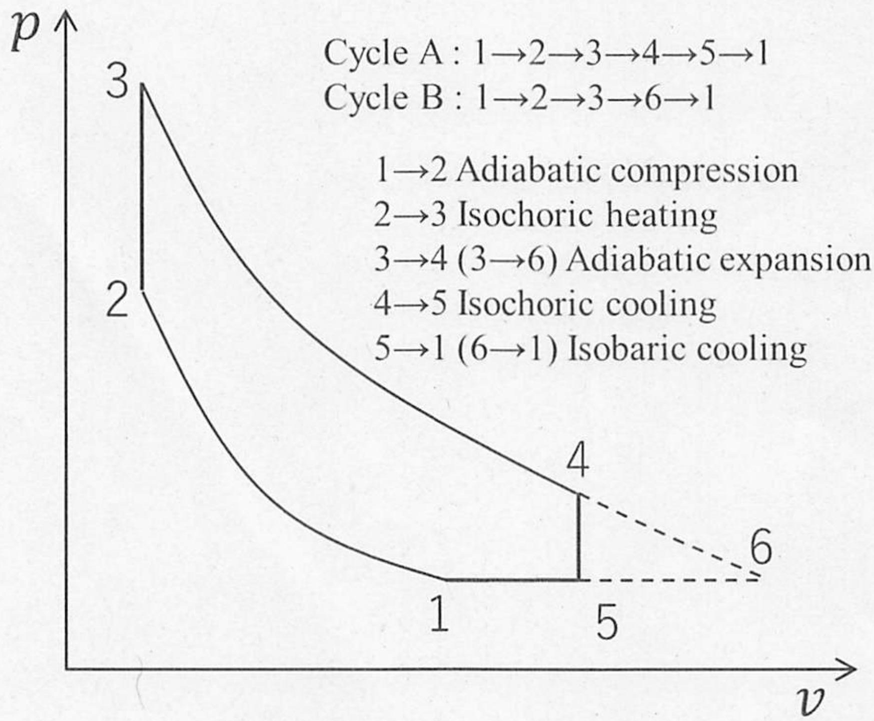

Figure 1